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Refining One Theorem For The Romanovsky Distribution

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ABSTRACT

The paper considered a refinement of the theorem for a negative-hypergeometric distribution(the Romanovsky distribution), i.e., convergence over variation of the Romanovsky distribution by Erlang distributions. The theorem is proved by the direct asymptotic method.

KEYWORDS

Negative hypergeometric distribution (the Romanovsky distribution), Erlang distribution, minimax problem.

INTRODUCTION

In probability theory, great emphasis has been placed on the study of asymptotic behaviors of different distributions. Among the many aspects of this problem, we will focus on one-the "minimax", like apparently originates from the work of Yu.V. Prokhorov [1] for the binomial distribution. As you know, the binomial distribution

$$B(k)=\begin{cases} C_n^k p^k (1-p)^{n-k}, k=0,1,2,\dots,n \\ 0\,, \quad k>n \end{cases}$$

Approximates in a certain sense with the normal or Poisson distributions

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$$\Pi_1(k) = \frac{(np)^k}{k!} e^{-np} \ , \ k = 0,1,2,\dots,$$

$$\Pi_2(k) = \frac{1}{\sqrt{2\pi npq}} e^{-\frac{u_k^2}{2}} \ , \ u_k = \frac{k-np}{\sqrt{npq}} \ , \ q = 1-p \ , \ k = 0,1,2,\dots,$$

$$\Pi_3(k) = \frac{(nq)^k}{k!} e^{-nq} \ , \ k = 0,1,2,\dots.$$

In [1], distance $\rho(B,\Pi_i) = \sum_{k=0}^{\infty} |B(k) - \Pi_i(k)|, \ i = 1,2,3$ was considered,

and minimax problem $\sup_{0\le p\le 1} \min_{i=1,2,3} \rho(B,\Pi_i) = \lambda n^{-\frac{1}{3}} + O(n^{-\frac{2}{3}})$ is solved, where $\lambda = 0,31295\dots$.

In the work of T.A. Azlarov, T.L. Gurvich, Prokhorov's results are refined in the following aspect

$$\overline{\Pi}_1(k) = \Pi_1(k)\left[1 + \frac{p}{2}(1-u_k^2)\right],$$

$$\overline{\Pi}_2(k) = \Pi_2(k)\left[1 + \frac{q-p}{6\sigma}\left(\frac{u_k^3}{q^{\frac{3}{2}}} - 3\frac{u_k}{q^{\frac{1}{2}}}\right)\right].$$

The minimax problem for this case has been solved: $\sup_{0\le p\le \frac{1}{2}} \min_i(\rho_i) = \overline{\lambda} n^{-\frac{2}{3}} + O(n^{-\frac{1}{3}})$, where

$\lambda = 0,26609\dots$.

The convergence in variation of the negative binomial distribution was studied by N. Arenbaev. The minimax problem for the hypergeometric distribution was solved by T.A. Azlarov, S.E. Umarov. Similar questions for negative geometric distribution (Romanovsky distribution) were solved by T.A. Azlarov and A.K. Yusupova in metric L_j.

SUPPORT SUGGESTIONS

V.I. Romanovsky [2] studied the following problem:

Let S_N, S_M be two ordered samples of volumes N and M:

$$x_1 \le x_2 \le \dots \le x_N \ ,$$
$$y_1 \le y_2 \le \dots \le y_M \ , \ \ (N \ge 1, M \ge 1)$$

from the same continuous collection S with a density of $f(x)$, which is unknown to us. In the first sample, highlighting the term x_{n+1} in it, we will have n members of it, which are not more than x_{n+1}, and $N-n-1$ members at least x_{n+1}. In this work of V.I. Romanovsky, the probability that the second sample will have μ members is not more than x_{n+1} and $M-\mu$ members over x_{n+1} :

$$P_n(k)=P(\mu=k)=R_{N,M,n}(k)=R(k)=\begin{cases}\dfrac{C_{n+k}^{n}C_{N+M-n-k-1}^{N-n-1}}{C_{N+M}^{N}}\ , & k=\overline{0,M}\\ 0\ , & k>M\end{cases} \tag{1}$$

$N\geq1$, $M\geq1$, $n=\overline{0,N-1}$.

V.I.. Romanovsky [2] proposes to use this distribution to construct a test for testing hypotheses about the homogeneity of the two samples under consideration and pointed out its very important applied side. It is for this reason and for brevity that we called the negative hypergeometric distribution (1) the Romanovsky distribution.

Studying the asymptotic behavior of the distribution $R(x)$ [5] - [7], we found that for various changes in the parameters, it approaches the normal, binomial, Poisson, negative binomial, Erlang and beta distributions.

Let

$$p=\frac{M}{N+M}\ ,\ q=\frac{N}{N+M}\ ,\ \alpha=\frac{n}{N}\ ,\ \beta=\frac{N-n}{N}$$

further, we introduce the notation for the distributions with which the Romanovsky distribution approaches.

$$\Pi_1(k)=\frac{\left(\frac{kq}{p}\right)^n\frac{q}{p}}{n!}\exp\left\{-\frac{kq}{p}\right\},\ k=0,1,2,\ldots,$$

$$\Pi_2(k)=\frac{\left(\frac{kq}{p}\right)^{N-n-1}\frac{q}{p}}{(N-n-1)!}\exp\left\{-\frac{kq}{p}\right\},\ k=0,1,2,\ldots$$

- Erlang distributions.

We also introduce $\rho(B,\Pi_i)=\sum_{k=0}^{\infty}\left|B(k)-\Pi_i(k)\right|,\ i=1,2$.

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In [3], it was proved that, for $M \to \infty$, $N \to \infty$, $p \to 1$, $\alpha \to 0$

$$\rho(R, \Pi_1) = \lambda\alpha + \alpha O\left(\min\left(1, \frac{q}{\sqrt{n+1}}\right)\right), \tag{2}$$

and for $M \to \infty$, $N \to \infty$, $p \to 1$, $\beta \to 0$

$$\rho(R, \Pi_2) = \lambda\beta + \beta O\left(\min\left(1, \frac{q}{\sqrt{N-n}}\right)\right), \tag{3}$$

where $\lambda = \frac{1}{2\sqrt{2\pi}} \int_{-\infty}^{\infty} \left|1-u^2\right| e^{-\frac{u^2}{2}} du$.

To clarify (2), (3), we introduce

$$\overline{\Pi}_1(k) = \Pi_1(k)\left[1 + \frac{\alpha+\beta}{2}(1-u_k^2)\right],$$

$$\overline{\Pi}_2(k) = \Pi_2(k)\left[1 + \frac{\beta+q}{2}(1-u_k^2)\right],$$

and consider $\rho(R, \overline{\Pi}_i) = \sum_{k=0}^{\infty} \left|R(k) - \overline{\Pi}_i(k)\right|$, $i = 1, 2$.

Convergence in variation of the Romanovsky distribution

Theorem. For $\alpha, q < \frac{1}{N}$

$$\rho(R, \overline{\Pi}_1) = \overline{\lambda}(\alpha+q) + (\alpha+q) O\left(\min\left(1, \max\left((\alpha+q)^2, \frac{1}{\alpha p(N+M)}\right)\right)\right),$$

a for $\beta, q < \frac{1}{N}$

$$\rho(R, \overline{\Pi}_2) = \overline{\lambda}(\beta+q) + (\beta+q) O\left(\min\left(1, \max\left((\beta+q)^2, \frac{1}{\beta p(N+M)}\right)\right)\right),$$

where $\overline{\lambda} = \frac{1}{8\sqrt{2\pi}} \int_{-\infty}^{\infty} \left|1-u^2\right| e^{-\frac{u^2}{2}} du$.

Proof. To prove the theorem, we use the following lemma.

Lemma. Let the bounded function $f(x)$ be integrable by $(-\infty, \infty)$ and have a finite number of l maxima and maxima. Let $x_j = j\Delta x + x_0$, where j takes integer values, $\Delta x > 0$. Then for any c and d.

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The proof of the lemma is given in [3].

Let 1, study 2 be divided into two parts, i.e. first we consider those values of 3 for which 4. In [4], the characteristic Erlang distribution function is given: 5.

With the help of 6 we have 7.

And easy calculations show that 8.

From (4) and (5) and from the condition of Theorem 9 we obtain 10.

Now consider those values 11 for which the sums in which the summation index satisfies inequality 12. Applying Stirling's formula and dividing easy calculations, we have 13.

It can be shown that 14.

It follows from (7) that 15.

Taking into account (6) and the fact that at 16 the Erlang distribution tends to the normal law, and applying the lemma to (8), we obtain the proof of the first part of the theorem. The second part of the theorem is proved in a similar way.

Further studies of similar theorems for the Romanovsky distribution will help refine the previously obtained minimax problem for this distribution.

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