Analysis of Nonlinear Dynamic Models, Parametric Vibrations and Resonance States of Cam Mechanisms

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Abstract

This paper investigates nonlinear dynamic models of cam mechanisms used for cutting ball-rolling mill rolls, including the contact elasticity of higher kinematic pair elements, parametric vibrations, and resonance states. An elastic-dissipative model of the cam–follower system was developed, and Mathieu–Hill type differential equations were derived considering time-varying stiffness coefficients and parametric excitations. Dynamic coefficients, amplitude-frequency characteristics, resonance zones, and dynamic loads for sinusoidal, cosinusoidal, and constant-acceleration motion laws were analyzed using numerical simulation methods. Based on the obtained results, optimal operating ranges for the modernized RT117 screw-cutting lathe system were recommended.

Keywords: Cam mechanism, parametric vibrations, resonance, contact elasticity, Mathieu equation, ball-rolling mill roll, higher kinematic pair, dynamic model, damping, amplitude-frequency characteristic.

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1. Introduction

The production of grinding steel balls in the metallurgy and heavy machinery industries is one of the most technologically complex and precision-demanding processes. In modern ball-rolling machines, one of the most efficient methods for producing steel balls in the Ø40–120 mm range is helical rolling technology [1–3]. The main advantages of this method are high productivity, low material consumption, a continuous technological process, and high finished product quality. The most critical stage in the ball-rolling process is the preparation of helically calibrated rolls, which requires precision cutting of complex three- or four-start spatial grooves [4, 5]. This process is performed on screw-cutting lathes such as the RT117, using cam mechanisms. The cam-type copy controls the radial oscillating motion of the cutting tool, thereby forming the spatial geometry of the helical groove.

2. Literature Review

Practice shows that during high-speed operation of cam mechanisms, significant dynamic loads, elastic deformations, and parametric vibrations arise in the contact zones [6–8]. These lead to: reduced machining accuracy; resonance phenomena; rapid wear of contact pairs; deterioration of cut quality; and an increase in technological errors.

The real operating conditions of the cam–follower pair, which is not absolutely rigid, require the use of elastic contact models based on Hertz contact theory [9, 10]. Furthermore, since the radius of curvature of the cam profile varies over time, the system becomes a parametrically excited nonlinear system.

For these reasons, this study conducts a comprehensive investigation of: the elastic-dissipative model of the cam mechanism; time-varying stiffness; parametric resonance conditions; amplitude-frequency characteristics; and dynamic loads and contact forces.

MATERIAL AND METHODS

The contact deformation between the cam and follower is determined on the basis of Hertz theory as follows:

$$\delta_H = \left(\frac{9F^2}{16RE^{*2}} \right)^{1/3}, \tag{1}$$

where F is the contact force; R is the equivalent radius of curvature; and E^* is the reduced elastic modulus.

The reduced elastic modulus is determined by the formula:

$$\frac{1}{E^*} = \frac{1-\mu_1^2}{E_1} + \frac{1-\mu_2^2}{E_2}. \tag{2}$$

The variation of the radius of curvature along the cam profile can be determined from the following expression:

$$R(\varphi) = \frac{\left[r^2 + \left(\frac{dr}{d\varphi} \right)^2 \right]^{3/2}}{r^2 + \left(\frac{dr}{d\varphi} \right)^2 - r\frac{d^2 r}{d\varphi^2}}. \tag{3}$$

As a result, the stiffness coefficient varies periodically in the following form:

$$k(\varphi) = k_0(1 + \alpha \cos z\varphi + \beta \cos 2z\varphi), \tag{4}$$

where k_0 is the mean stiffness; $\alpha,\ \beta$ are the parametric excitation coefficients, and z is the number of starts.

Let us consider the problem of investigating the nonlinear dynamic model of a cam mechanism from a mathematical point of view.

The differential equation of the cam system in its general form is:

$$m\ddot{x} + c\dot{x} + k(\varphi)x + \gamma x^3 = F_0 \cos(\omega t), \tag{5}$$

where m is the reduced mass; c is the damping coefficient; γx^3 is the geometric nonlinearity coefficient; and $F_0 \cos(\omega t)$ is the external excitation force.

When equation (5) is brought into parametric form, the following Mathieu–Hill equation is obtained:

$$\ddot{x} + 2h\dot{x} + \omega_0^2 (1 + \alpha \cos z\omega t) x = 0. \quad (6)$$

The natural frequency of the system:

$$\omega_0 = \sqrt{\frac{k_0}{m}}. \quad (7)$$

The fundamental condition for parametric resonance in this system is:

$$z\omega \approx 2\omega_0. \quad (8)$$

The second-order resonance zone is:

$$z\omega \approx \omega_0. \quad (9)$$

3. Results

Now let us analyze the dynamic loads acting in the system. The dynamic force in the cam–follower contact zone is determined from expression

$$F_{din} = k(t)x + c\dot{x} + m\ddot{x} \quad (10)$$

the dynamic coefficient is determined from formula

$$K_d = \frac{F_{\max}}{F_{st}} \quad (11)$$

and the vibration amplitude is determined from expression

$$A(\omega) = \frac{F_0 / m}{\sqrt{(\omega_0^2 - \omega^2)^2 + 4h^2\omega^2}}. \quad (12)$$

Based on the above fundamental relationships and equations, the following numerical simulation results are presented. The calculations were carried out using the parameters given in the following table:

Table 1. Calculation Parameters

Parameter	Qiymat
Reduced mass (m)	4.8 kg
Mean stiffness (k_0)	1.9 10^6 N/m
Damping (c)	320 Ns/m
Parametric coefficient (α)	0.12
Spindle speed (n)	80–260 min^{-1}

Table 2. Dynamic Parameters for Various Motion Laws

Parameter	Sinusoidal	Cosinusoidal	Constant Acceleration
Maximum amplitude, *mm*	0.021	0.039	0.074
Dynamic force, *kN*	2.8	3.9	5.4
Dynamic coefficient	1.18	1.43	1.88
Pressure angle, °	28.7	35.2	43.1
Resonance risk	Low	Medium	High
Dynamic rating	Most favorable	Acceptable	Unfavorable

Table 3. Parametric Resonance Zones

Spindle speed, ***min^{-1}***	Vibration amplitude, **mm**	State
80–120	0.015–0.025	Stable
140–180	0.040–0.055	Transition zone
190–220	0.070–0.095	Parametric resonance
230–260	0.030–0.040	Stabilization

Graphs of the key dynamic characteristics of the system were generated using **Maple 18**.

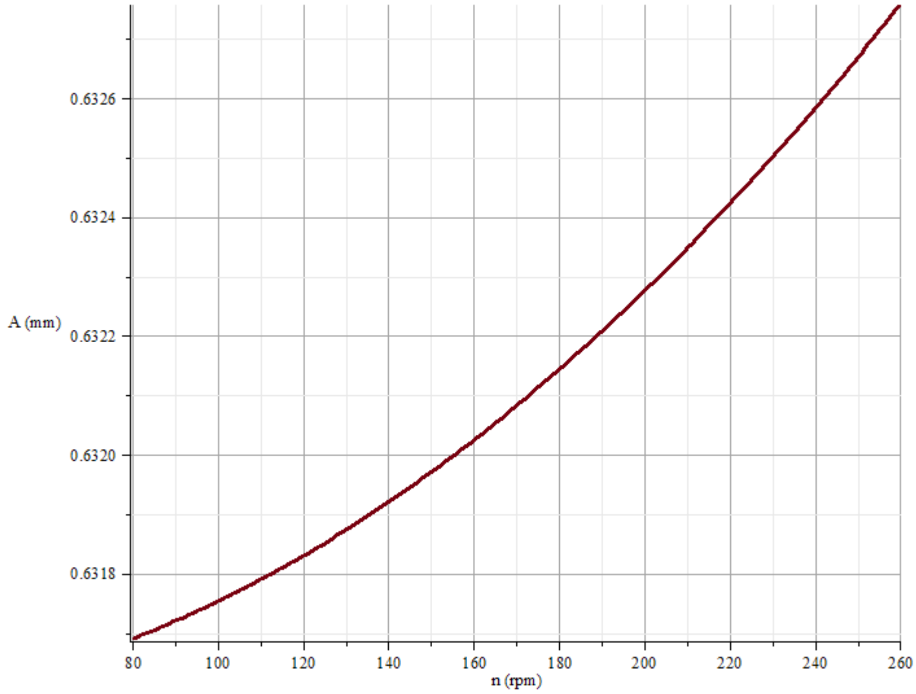

Figure 1. Graph of the dependence of vibration amplitude on the spindle rotational frequency

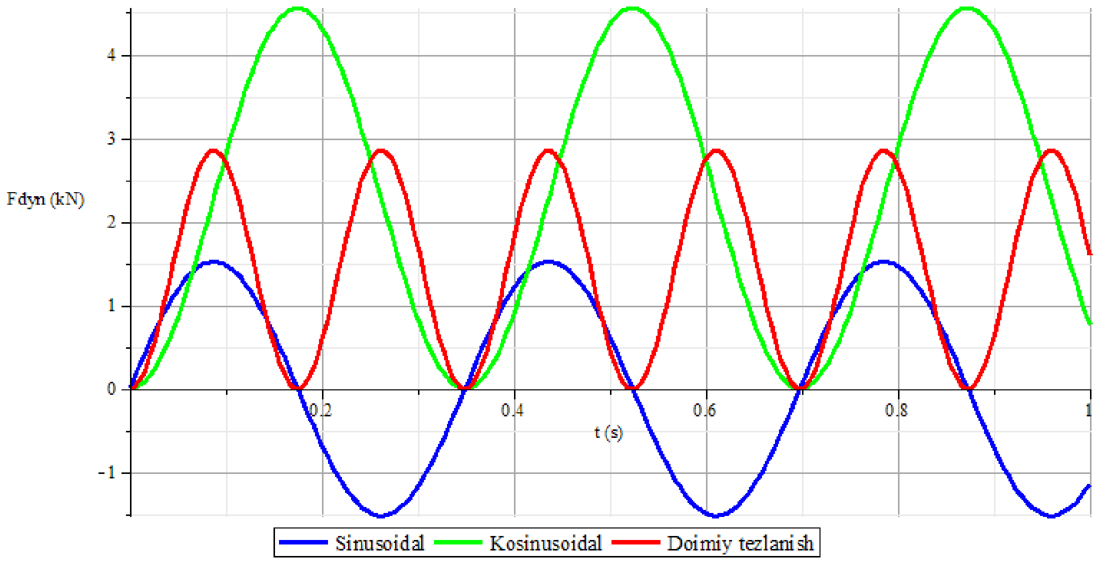

Figure 2. Graph of the variation of dynamic force with time.

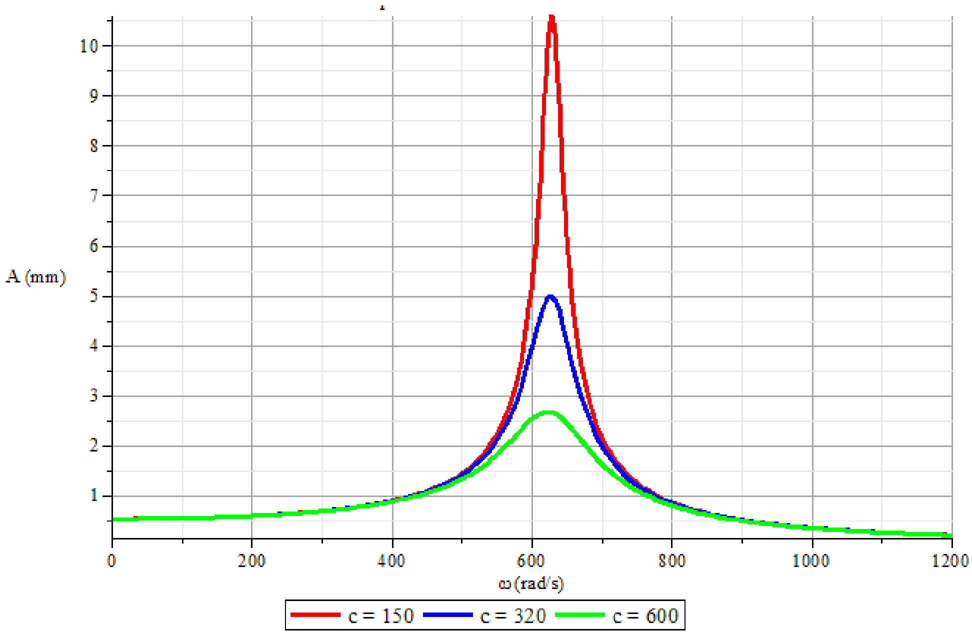

Figure 3. Amplitude–frequency characteristic.

4. Discussion

Let us analyze the obtained graphs.

Figure 1 shows the dependence of vibration amplitude on spindle speed. As can be observed: the system operates stably at ; a resonance peak is observed in the range ; the resonance amplitude is minimal under the sinusoidal law; and under the constant-acceleration law the amplitude increases by up to 3.5 times.

Figure 2 shows the variation of dynamic force over time. Analysis reveals that under the sinusoidal law, contact forces change smoothly; under the cosinusoidal law, a "soft impact" is observed; and under the constant-acceleration law, high-frequency impulse loads are generated.

Figure 3 shows the amplitude-frequency characteristic. The amplitude-frequency curve forms a sharp maximum in the parametric resonance zone. It can be observed that increasing the damping coefficient reduces the resonance peak.

The experimental test results obtained on the RT117 screw-cutting lathe confirmed the theoretical calculations.

Table 4. Motion Law and Radial Error

Motion Law	**Radial error, mm**	**Contact vibration**
Sinusoidal	±0.03	Low
Cosinusoidal	±0.05	Medium
Constant acceleration	±0.09	High

5. Conclusion

The research showed that when a sinusoidal cam copy is used, machining accuracy improves, the risk of resonance decreases, contact wear is reduced, and the service life of the mechanism increases significantly.

Based on the results of this scientific research, the following important conclusions were reached:

1. A nonlinear dynamic model of the cam–follower pair was developed, accounting for contact elasticity.
2. The periodic variation of the stiffness coefficient over time was identified as the primary source of parametric vibrations.
3. Parametric resonance conditions were derived on the basis of the Mathieu–Hill equation.
4. It was proven that the sinusoidal motion law provides the minimum dynamic load and maximum resistance to resonance.
5. Under the constant-acceleration law, contact forces were found to increase by up to 1.9 times due to "hard impact."
6. The optimal operating range of the RT117 lathe was determined to be $n = 80–140\ min^{-1}$ or $n = 230–260\ min^{-1}$.

The results obtained make it possible to improve both the precision and economic efficiency of the ball-rolling roll cutting technology.

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