Mathematical Model Of Closed-Loop Aerodynamic Tube

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Abstract

This study explores a closed-loop aerodynamic tube that recirculates air to create stable, repeatable flow conditions in a laboratory setting. In this configuration, a fan continuously drives air through a closed duct, past a central test section where measurements and experiments are carried out, and then back to the fan inlet. The closed loop reduces the impact of ambient disturbances and helps maintain the desired flow with relatively modest energy use. The test section is designed with a nearly constant cross-sectional area so that the velocity field is as uniform as possible, which simplifies both instrumentation and data interpretation. Upstream, a contraction accelerates and conditions the flow, while downstream diffusers and curved ducts gently slow and redirect the air, preserving flow quality as it recirculates. Taken together, these features provide a compact and efficient platform for high-fidelity aerodynamic testing and control-oriented studies of internal flows.

Keywords: losed-loop aerodynamic tube, wind tunnel, recirculating airflow, test section design, flow conditioning, contraction-diffuser system, laboratory aerodynamics.

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1. Introduction

Closed-loop aerodynamic tubes are commonly employed in experimental aerodynamics, flow metrology, and sensor calibration because they can create controlled and repeatable airflow conditions in a small, energy-efficient setup. Closed-loop wind tunnels, on the other hand, recirculate the working fluid [1]. This makes the flow more stable, less sensitive to changes in the environment, and requires less power to run. Because of these features, closed-loop aerodynamic tubes are great for testing and calibrating airflow measurement tools in the lab [2].

Even though they are used in many different ways, closed-loop aerodynamic tubes are often designed and built using empirical rules and steady-state performance curves [3]. While these methods work well for early design, they don't give us much information about transient behavior, nonlinear flow dynamics, or how the fan and the aerodynamic circuit work together. As a result, getting flow control to work right and consistently often requires careful tuning and a lot of trial and error, especially when the conditions change [4].

From the perspective of systems theory, a closed-loop aerodynamic tube constitutes a dynamic system in which airflow velocity is dictated by the balance between pressure losses due to friction, geometric transitions, and flow-conditioning elements, and the pressure produced by the fan. These effects are nonlinear by nature, and they depend on the instantaneous flow state and system characteristics. Without a reliable mathematical model, it is hard to predict how a system will respond, make control plans, or evaluate design changes[5].

In this study, a mathematical model of a closed-loop aerodynamic tube is developed to characterize its dynamic behavior in a manner that is both manageable and physically intelligible. The model takes into account the unique behavior of the driving fan, the pressure losses along the flow channel, and the inertia of the airflow [6,7]. The proposed approach simplifies the development of feedback control systems for aerodynamic testing and calibration, enabling efficient simulation through the condensation of governing equations.

PHYSICAL DESCRIPTION OF THE CLOSED-LOOP AERODYNAMIC TUBE AND MODELING ASSUPTIONS.

The closed-loop aerodynamic tube used in this study is a recirculating airflow system that provides stable, repeatable flow conditions in a controlled laboratory setting. Air is continuously driven by a fan through a closed duct, passes through the test section where measurements or experiments are carried out, and then returns to the fan inlet. This recirculating layout reduces the impact of changing ambient conditions and makes it possible to maintain the airflow with relatively low energy use [8,9].

The test section is the core of the system and is designed to create a well-defined flow field. It typically has a constant cross-sectional area to achieve an almost uniform velocity profile and to make both measurements and data analysis easier. Upstream of the test section, a contraction section is usually installed to accelerate the flow and improve its quality, while downstream components such as diffusers and curved ducts gradually slow the air down and guide it back toward the fan [10].

To make the flow more uniform, the loop often includes flow-conditioning devices such as honeycombs or screens [11]. These components help remove swirl and large-scale velocity variations caused by the fan or by changes in duct geometry, but they also introduce extra pressure losses that influence how the system behaves as a whole. Similar losses occur in bends, expansions, and contractions along the duct, all of which dissipate energy and limit the maximum airflow velocity that can be reached [12].

To keep the mathematical description of the system tractable while still realistic, several simplifying assumptions are made [13]. The air is treated as incompressible, which suits the low-speed operating conditions typical of laboratory aerodynamic tubes, so air density is taken as constant and thermal effects are neglected, meaning temperature changes do not significantly affect the flow. In each cross section, the velocity is represented by its average value, so the airflow can be modeled with a one-dimensional, lumped-parameter approach [14].

Figure 1 presents a closed-loop aerodynamic tunnel with an open test section located between the upstream and downstream duct segments. The tunnel comprises a series of straight and curved sheet-metal ducts mounted on a rigid support frame, which together form a recirculating flow path for the air. The open region in the middle of the layout provides experimental access to the test section, allowing models or equipment to be installed directly in the core flow while the remaining loop preserves the overall flow continuity and operating conditions.

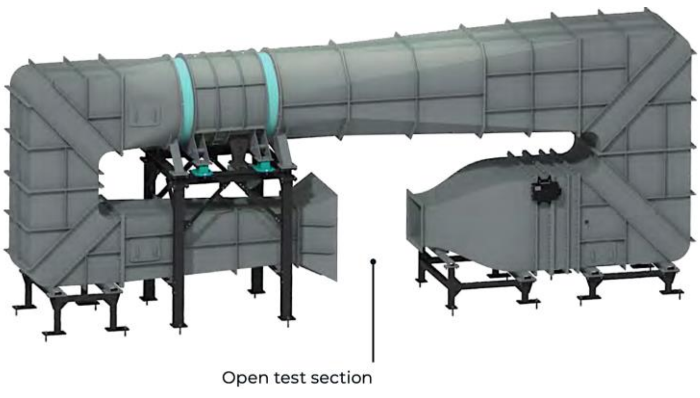

Figure 1. An example of closed-loop aerodynamic tunnel [15]

GOVERNING EQUATIONS OF AIRFLOW IN THE CLOSED-LOOP AERODYNAMIC TUBE.

The way the closed-loop aerodynamic tube behaves over time is determined by how the airflow velocity changes along the recirculating duct. Using the assumptions introduced earlier, this airflow can be described with a one-dimensional, lumped-parameter model, where the spatially distributed flow is represented by its average velocity in the test section. This simplified description retains the key physical effects while keeping the mathematics manageable and well suited for developing and analyzing control strategies.

The continuity equation says that the volumetric flow rate must stay the same at every point in the closed loop for an incompressible flow where the air density doesn't change. The amount of air that flows through any cross-section of the tunnel at any given time stays the same throughout the system. This condition can be expressed as

$$Q = A v$$

Where Q is the volumetric flow rate,A is the cross-sectional area of the test section, and v is the average airflow velocity.

Since the loop is closed and no mass enters or leaves the system, the same flow rate circulates through all components of the tube.

The balance between the fan's pressure rise and the total pressure losses that build up along the flow path sets the rate at which the airflow changes over time. The fan pushes the flow, but friction and other losses work against it. By using a momentum balance on the effective mass of air in the loop, this trade-off can be turned into a governing equation for the speed of the airflow:

$$\rho AL(dv/dt) = \Delta p_{fan} - \Delta p_{losses}$$

Where ρ is the air density, L is the effective flow path of the aerodynamic tube, Δp_{fan} is the pressure rise produced by the fan and Δp_{losses} represents the total pressure losses.

The term on the left-hand side represents the inertia of the moving air and emphasizes that changes in velocity cannot happen instantly. Instead, the airflow adjusts gradually over time, responding dynamically to any pressure imbalance in the loop.

Pressure losses in the closed-loop aerodynamic tube arise from two main sources: friction distributed along the straight duct sections and local losses introduced by geometric elements such as bends, contractions, diffusers, screens, honeycombs, and corner vanes. Distributed friction losses are described using the Darcy–Weisbach formulation, which provides a convenient way to relate pressure drop to flow velocity, duct length, and hydraulic characteristics:

$$\Delta p_f = \lambda(L/D)(\rho v^2/2)$$

Where λ is the Darcy friction factor and D is the diameter of the duct.

Local losses are expressed as

$$\Delta p_l = \Sigma\zeta_i(\rho v^2/2)$$

Where ζ_i denotes the loss coefficient associated to the i-th local element.

The total pressure loss is therefore given by

$$\Delta p_{losses} = \Delta p_f + \Delta p_l$$

This formulation explicitly highlights the quadratic dependence of pressure losses on airflow velocity, which introduces a strong nonlinearity into the system dynamics.

The fan supplies the driving force needed to keep the air circulating in the closed loop. Its pressure rise is usually described by a characteristic curve that links the generated pressure to the airflow velocity or volumetric flow rate. In a simplified form, this fan characteristic can be written as

$$\Delta p_{fan} = a^0 - a^1 v - a^2 v^2$$

where a^0, a^1, and a^2 are fan-specific coefficients determined from manufacturer data or from experiments. This representation captures how the available pressure rise decreases as the airflow velocity increases and is convenient for dynamic simulations and control design.

By combining the expressions for airflow inertia, fan-induced pressure rise, and pressure losses, the closed-loop aerodynamic tube can be described by a single nonlinear differential equation. In compact form, this reads

$$\rho AL(dv/dt) = a^0 - a^1 v - a^2 v^2 - (\lambda L/D + \Sigma\zeta_i)(\rho v^2/2)$$

This equation constitutes the core mathematical model of the system and offers a clear physical picture of how airflow velocity changes under competing mechanisms of pressure generation and dissipation. Because of its compact structure, the model is well suited to numerical simulation, parameter identification, and the design of feedback control strategies.

CONTROL-ORIENTED FORMULATION AND SYSTEM REPRESENTATION

The mathematical model developed in the previous section offers a physically consistent description of the airflow dynamics in the closed-loop aerodynamic tube. To enable analysis, simulation, and controller design, this model is now recast in a control-oriented form, which emphasizes the role of the control input, clarifies the system dynamics, and supports the use of modern feedback control methods.

The main dynamic quantity of interest is the airflow velocity in the test section, because it directly sets the operating conditions of the tube. Consequently, the system state is defined as

x = v

where v is the average airflow velocity in the test section.

The control input is linked to the fan actuation, which may be implemented through motor voltage, rotational speed, or a normalized command such as a PWM duty cycle. For generality, the control input is written as

$$u = u_f$$

where u_f denotes the fan actuation command.

Using the compact dynamic model introduced above, the airflow dynamics can be written in state-space form as

$$dv/dt = f(v, u)$$

Substituting the expressions for the fan-generated pressure and the pressure losses gives

$$dv/dt = \big(1/(\rho AL)\big)\big[\Delta p_{fan}(u, v) - \Delta p_{losses}(v)\big]$$

Or, explicitly,

$$dv/dt = \big(1/(\rho AL)\big)[a^0(u) - a^1 v - a^2 v^2 - (\lambda L/D + \Sigma\zeta_i)(\rho v^2/2)]$$

In most practical setups, the airflow velocity in the test section is either directly measured or reliably estimated using standard flow sensors. Therefore, the system output is simply defined as

y = v

which streamlines controller design and enables the measured velocity to serve directly as feedback for regulation or tracking tasks.

For controller design and stability analysis, it is often useful to linearize the nonlinear model around a nominal operating point characterized by a steady-state airflow velocity v_0 and corresponding control input u_0. Linearization leads to

$$d(\delta v)/dt = A\delta v + B\delta u$$

Where $\delta v = v - v_0$ and $\delta u = u - u_0$.

The linearized system matrices are given by

$$A = (\partial f/\partial v)|_{(v^0, u^0)}$$

$$B = (\partial f/\partial u)|_{(v^0, u^0)}$$

The control-oriented formulation brings out several key features of the closed-loop aerodynamic tube. It reveals pronounced nonlinear behavior due to the quadratic dependence of pressure losses on airflow velocity, a dynamic lag introduced by airflow inertia that limits how fast the system can respond, and a strong sensitivity of model parameters to geometric details and flow-conditioning devices, which may change with operating conditions or configuration adjustments.

Together, these characteristics motivate the use of feedback control strategies that can tolerate parameter uncertainty while maintaining stable and accurate airflow regulation over a broad operating range. The resulting model forms a solid foundation for designing and assessing control algorithms, which are crucial for achieving high-precision airflow control in aerodynamic testing and calibration tasks.

2. Conclusion

The study has developed a control-oriented dynamic model of a closed-loop aerodynamic tube that links airflow velocity in the test section to fan actuation and distributed and localized pressure losses. The resulting nonlinear lumped-parameter formulation provides a compact but physically meaningful description of the dominant flow phenomena, including inertia-induced response lag and the quadratic dependence of losses on velocity. This structure proved convenient for recasting the system in state-space form, defining appropriate state and control variables, and deriving both nonlinear and locally linearized models suitable for feedback controller design and analysis.

At the same time, the modeling framework has several limitations that should be acknowledged. The assumption of incompressible flow with constant density restricts the applicability of the model to low-speed operating regimes and precludes direct extension to high-Mach or strongly heated flows. Likewise, the one-dimensional, spatially averaged representation cannot capture secondary flow structures, boundary-layer development, or three-dimensional non-uniformities in the test section that may become important for complex test articles or high blockage ratios. Model parameters such as friction factors and local loss coefficients are also treated as fixed, even though they may vary with Reynolds number, surface roughness, and configuration changes in screens, honeycombs, or corner vanes.

These constraints open several promising directions for future research. A natural extension is to incorporate spatially distributed effects-either through higher-order lumped models or reduced-order models derived from computational fluid dynamics or detailed experimental data-to better describe non-uniform and unsteady flow features in the test section. Another avenue is the systematic identification of operating-point-dependent loss and fan characteristics, enabling gain-scheduled or adaptive controllers that remain robust under configuration changes and parameter drift. Finally, integrating the present model with advanced control strategies, such as robust, model-predictive, or data-driven controllers, and validating them in hardware-in-the-loop or experimental wind-tunnel campaigns would help close the gap between theoretical development and high-precision aerodynamic testing practice.

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